Questions

Example Find the most general antiderivative for the function

\[ g(x) = \frac{5 - 4x^3 + 2x^6}{x^6} \]

Example Find \( f \) when

\[ f'(x) = 2x - 3/x^4, \quad x > 0, \quad f(1) = 3. \]

Example Find \( f \) when

\[ f''(x) = 4 - 6x - 40x^3, \quad f(0) = 2, \quad f'(0) = 1. \]

Example A particle is moving with the given data. Find the position of the particle.

\[ v(t) = 1.5\sqrt{t}, \quad s(4) = 10. \]

Example A particle is moving with the given data. Find the position of the particle.

\[ a(t) = \cos t + \sin t, \quad s(0) = 0, \quad v(0) = 5. \]

Example A car is traveling at 50 mph when the brakes are fully applied, producing a constant deceleration of 22 ft/s². What is the distance covered before the car comes to a stop?

Solutions

Example Find the most general antiderivative for the function

\[ g(x) = \frac{5 - 4x^3 + 2x^6}{x^6} \]

We cannot find an antiderivative of the function in its present form, so we should use algebra to rewrite the function in a form for which we can get the antiderivative.

\[ g(x) = \frac{5 - 4x^3 + 2x^6}{x^6} \]
\[ = \frac{5x^{-6} - 4x^{-3} + 2}{x^6} \]
\[ G(x) = 5 \frac{1}{-5} x^{-5} - 4 \frac{1}{-2} x^{-2} + \frac{1}{1} 2x + C \]
\[ = \frac{-x^{-5}}{5} + \frac{2}{2} x^{-2} + 2x + C \]
\[ G'(x) = -(5)x^{-6} + 2(-2)x^{-3} + 2(1) \]
\[ = 5x^{-6} - 4x^{-3} + 2 \]
\[ = g(x) \]
Example Find $f$ when

$$f'(x) = 2x - 3/x^4, \ x > 0, f(1) = 3.$$ We can get $f$ with a single antidifferentiation. Then we will use the condition $f(1) = 3$ to determine the constant that is introduced.

\[
\begin{align*}
  f'(x) &= 2x - 3x^{-4} \\
  f(x) &= 2 \cdot \frac{1}{2} x^2 - 3 \cdot \frac{1}{3} x^{-3} + C \\
       &= x^2 + x^{-3} + C \\
  f(1) &= (1)^2 + (1)^{-3} + C \\
       &= 2 + C \\
  f(1) &= 3 \\
  3 &= 2 + C \\
  1 &= C \\
  f(x) &= x^2 + x^{-3} + 1
\end{align*}
\]

Example Find $f$ when

$$f''(x) = 4 - 6x - 40x^3, f(0) = 2, f'(0) = 1.$$ We can get $f$ with two antidifferentiations. Then we will use the two conditions to determine the constants that are introduced.

\[
\begin{align*}
  f''(x) &= 4 - 6x - 40x^3 \\
  f'(x) &= 4x - 6 \cdot \frac{1}{2} x^2 - 40 \cdot \frac{1}{4} x^4 + C_1 \\
       &= 4x - 3x^2 - 10x^4 + C_1 \\
  f(x) &= 4 \cdot \frac{1}{2} x^2 - 3 \cdot \frac{1}{3} x^3 - 10 \cdot \frac{1}{5} x^5 + C_1 x + C_2 \\
       &= 2x^2 - x^3 - 2x^5 + C_1 x + C_2 \\
  f'(0) &= 4(0) - 3(0)^2 - 10(0)^4 + C_1 \\
       &= C_1 \\
  f'(0) &= 1 \\
  C_1 &= 1 \\
  f(0) &= 2(0)^2 - (0)^3 - 2(0)^5 + (0) + C_2 \\
       &= C_2 \\
  f(0) &= 2 \\
  C_2 &= 2 \\
  f(x) &= 2x^2 - x^3 - 2x^5 + x + 2
\end{align*}
\]
Example A particle is moving with the given data. Find the position of the particle.

\[ v(t) = 1.5\sqrt{t}, \ s(4) = 10. \]

\[
\begin{align*}
  v(t) &= 1.5\sqrt{t} = \frac{3}{2} t^{1/2} \\
  s(t) &= \frac{3}{2} \cdot \frac{1}{3/2} t^{3/2} + C \\
        &= t^{3/2} + C \\
  s(4) &= 4^{3/2} + C \\
        &= 8 + C \\
  s(4) &= 10 \\
  8 + C &= 10 \\
  C &= 2 \\
  s(t) &= t^{3/2} + 2
\end{align*}
\]

Example A particle is moving with the given data. Find the position of the particle.

\[ a(t) = \cos t + \sin t, \ s(0) = 0, \ v(0) = 5. \]

\[
\begin{align*}
  a(t) &= \cos t + \sin t \\
  v(t) &= \sin t - \cos t + C_1 \\
  s(t) &= -\cos t - \sin t + C_1 t + C_2 \\
  v(0) &= \sin 0 - \cos 0 + C_1 \\
        &= 0 - 1 + C_1 \\
        &= -1 + C_1 \\
  v(0) &= 5 \\
  -1 + C_1 &= 5 \\
  C_1 &= 6 \\
  s(0) &= -\cos 0 - \sin 0 + 6(0) + C_2 \\
        &= -1 - 0 + C_2 \\
        &= -1 + C_2 \\
  s(0) &= 0 \\
  -1 + C_2 &= 0 \\
  C_2 &= 1 \\
  s(t) &= -\cos t - \sin t + 6t + 1
\end{align*}
\]
Example A car is traveling at 50 mph when the brakes are fully applied, producing a constant deceleration of 22 ft/s\(^2\). What is the distance covered before the car comes to a stop?

The first thing we have to do with this problem is make sure the units are consistent. I choose to work with feet and seconds.

1 h = 60 min = 3600 s.
1 mile = 5280 feet.
50 mph = \(50 \text{ miles \ hour} = 50 \cdot \frac{5280 \text{ feet}}{3600 \text{ s}} = 220/3 \text{ ft/s}\).

Now we can start to solve the problem. We know the velocity is \(220/3 \text{ ft/s}\) when the brakes are applied, and we know the acceleration during the braking period is \(-22 \text{ ft/s}^2\) (negative because the car is slowing down). We have

\[ a(t) = -22 \]
\[ v(t) = -22t + C_1 \]
\[ s(t) = -22 \cdot \frac{1}{2} t^2 = -11t^2 + C_1 t + C_2 \]

Using the initial velocity as \(220/3 \text{ ft/s}\), we can determine \(C_1\) (we assume that \(t = 0\) is when the brakes are applied):

\[ v(t) = -22t + C_1 \]
\[ v(0) = -22(0) + C_1 = 220/3 \]
\[ v(t) = -22t + 220/3 \]

and the position is given by:

\[ s(t) = -11t^2 + \frac{220}{3} t + C_2 \]

The distance it takes the car to stop is given by

\[ s(t_s) - s(0) = -11t_s^2 + \frac{220}{3} t_s + C_2 + 11(0)^2 - \frac{220}{3}(0) - C_2 = -11t_s^2 + \frac{220}{3} t_s \]

where \(t_s\) is the time it takes for the car to stop. Notice that we do not need to know the value of \(C_2\) to solve the problem!

The time it takes the car to stop is determined by when the velocity is zero,

\[ v(t_s) = 0 = -22t_s + 220/3, \]

which leads to \(t_s = 10/3\).

The distance it takes the car to stop is

\[ s(10/3) - s(0) = -11 \left(\frac{10}{3}\right)^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.22 \text{ ft.} \]

This number is called the *braking distance* on the following UK website [http://www.hintsandthings.co.uk/garage/stopmph.htm](http://www.hintsandthings.co.uk/garage/stopmph.htm), and they estimate a braking distance of 125 feet when traveling at 50 mph.