

Graphical Transformations

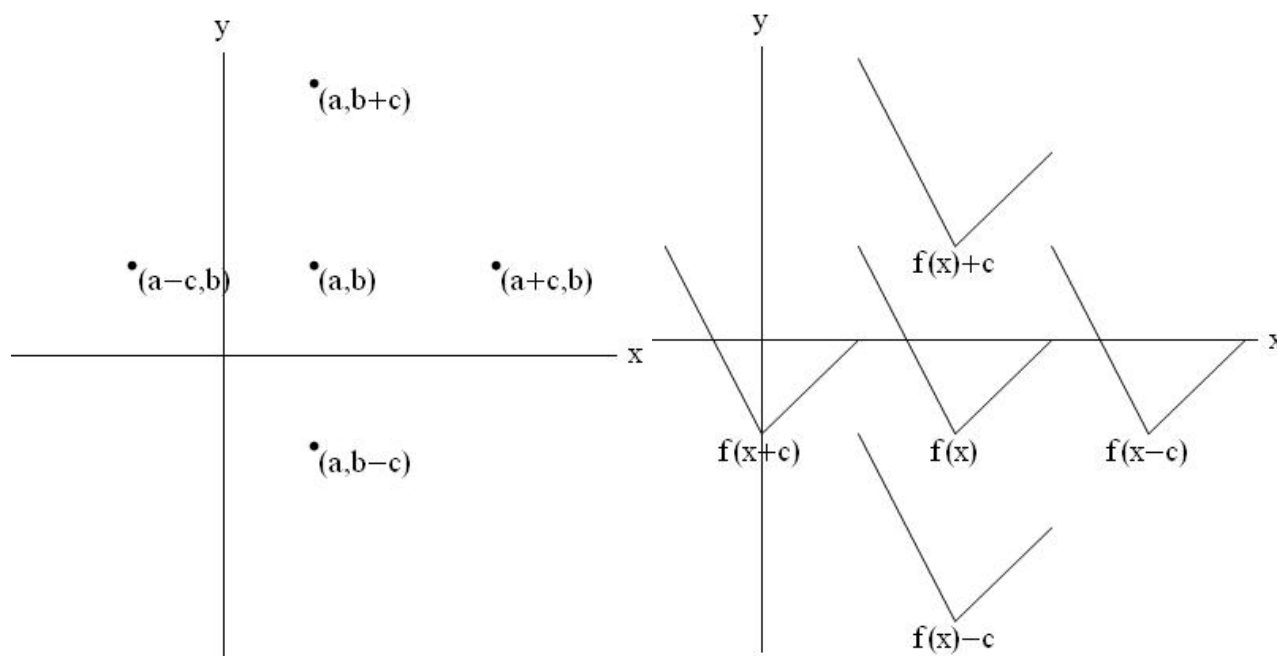
If we take a graph of a function and apply transformations to it, we can quickly determine the graph of a wide variety of related functions.

This is important since

- it allows us to obtain quickly sketches of complicated functions,
- it helps reinforce the properties of the basic functions from which the more complicated functions are built,
- gives us practice with functional notation.

Vertical and Horizontal Shifts:

Suppose $c > 0$. Here is how the ordered pair (a, b) is moved by adding or subtracting $c > 0$:



If $y = f(x)$ is our original function, the ordered pairs we have are $(x, y) = (x, f(x))$.

We can transform this function in the following ways:

- A shift up of c units yields the ordered pairs $(x, f(x) + c)$ which means $y = f(x) + c$.
- A shift down of c units yields the ordered pairs $(x, f(x) - c)$ which means $y = f(x) - c$.
- A shift right of c units yields the ordered pairs $(x + c, f(x)) = (x, f(x - c))$ which means $y = f(x - c)$.
- A shift left of c units yields the ordered pairs $(x - c, f(x)) = (x, f(x + c))$ which means $y = f(x + c)$.

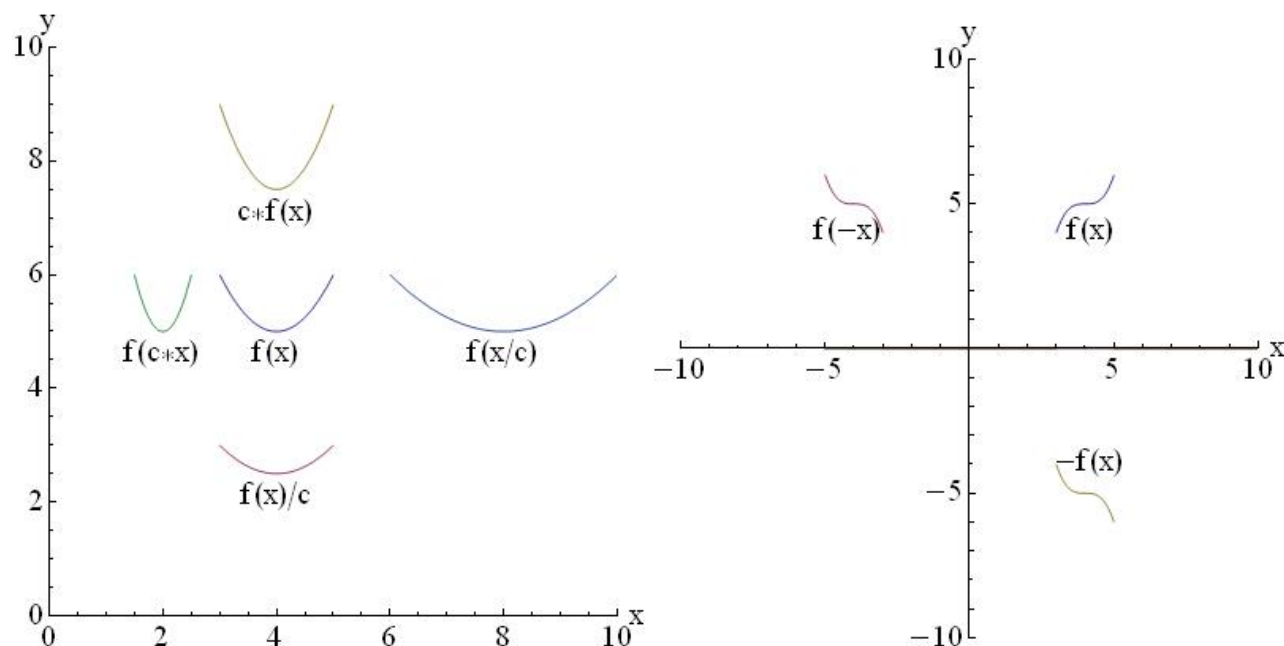
Vertical and Horizontal Stretching and Reflecting:

Stretching: If $c > 1$ then the graph of $y = cf(x)$ is the graph of $y = f(x)$ stretched by a factor of c in the vertical direction.

Reflection: The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x -axis, because our ordered pair has changed from $(x, f(x))$ to $(x, -f(x))$.

Suppose $c > 1$. Then, to obtain the graph of

- $y = cf(x)$ stretch the graph of $y = f(x)$ vertically by a factor of c ,
- $y = (1/c)f(x)$ compress the graph of $y = f(x)$ vertically by a factor of c ,
- $y = f(cx)$ compress the graph of $y = f(x)$ horizontally by a factor of c ,
- $y = f(x/c)$ stretch the graph of $y = f(x)$ horizontally by a factor of c ,
- $y = -f(x)$ reflect the graph of $y = f(x)$ about the x -axis,
- $y = f(-x)$ reflect the graph of $y = f(x)$ about the y -axis.



NOTE: The stretching looks like translations, but notice how the shape of the graph changes. The translations and reflections are rigid transformation because the shape of the graph is not changed. Stretching and compressing are nonrigid transformations since the shape of the graph is changed.

You can combine these transformations, but be careful to remember that the order in which these transformations are performed can sometimes matter!