Ex $f(x) = 3x+2$ Find $(f \circ g)(x)$ and it's domain.
$g(x) = x^2 - 2$ Find $(g \circ f)(x)$ and it's domain.

$$(f \circ g)(x) = f(g(x))$$
$$= f(x^2 - 2)$$
$$= 3(x^2 - 2) + 2$$
$$= 3x^2 - 4.$$

domain of f $(-\infty, \infty)$

range of f $(-\infty, \infty)$

domain of g $(-\infty, \infty)$

range of g $[-2, \infty)$

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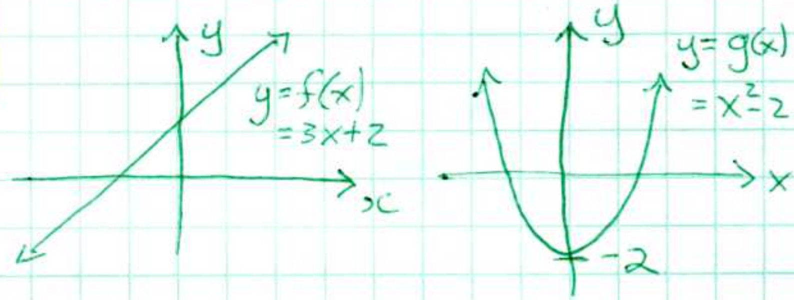
sketches (if needed)



To get domain of $f \circ g$:

domain of g $(-\infty, \infty)$

range of g $[-2, \infty)$

domain of f $(-\infty, \infty)$

→ intersection of these sets is $[-2, \infty)$. Since the range of g is entirely contained in the domain of f, we can use the entire domain of g as the domain of $f \circ g$. Done!
Domain of $f \circ g$ is $(-\infty, \infty)$.

$$(g \circ f)(x) = g(f(x))$$
$$= g(3x+2)$$
$$= (3x+2)^2 - 2$$
$$= 9x^2 + 12x + 2$$

To get domain of $g \circ f$:

domain of f: $(-\infty, \infty)$

range of f is $(-\infty, \infty)$

domain of g is $(-\infty, \infty)$

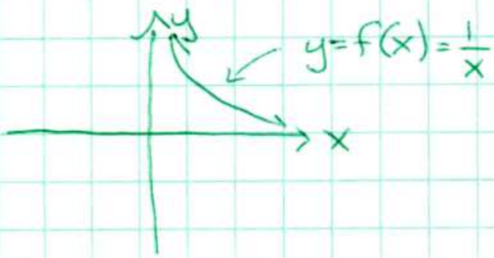
→ intersection of these sets is $(-\infty, \infty)$. Therefore, we can use the entire domain of f as the domain of $g \circ f$.

Domain of $g \circ f$ is $(-\infty, \infty)$.

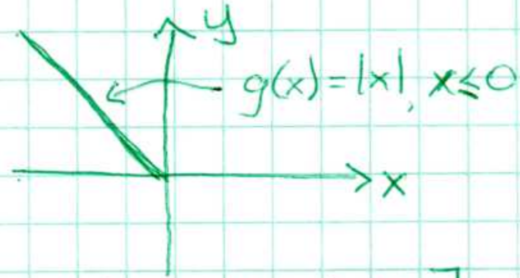
Ex/ $f(x) = \frac{1}{x}$, $x > 0$.

$g(x) = |x|$, $x \leqslant 0$.

Find $f \circ g$ and domain.
Find $g \circ f$ and domain.



domain is $(0, \infty)$
range is $(0, \infty)$



domain is $(-\infty, 0]$
range is $[0, \infty)$

Note: You can simplify
$g(x) = |x| = -x$
since $x \leqslant 0$.

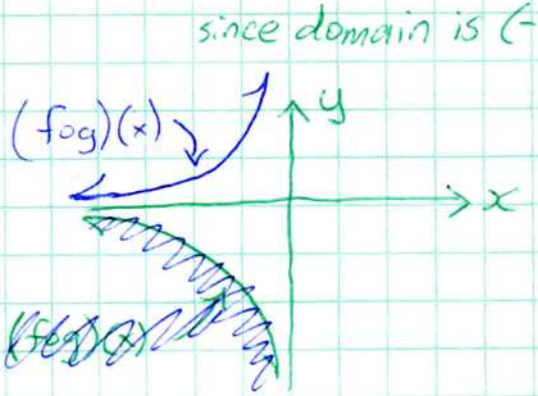
$$(f \circ g)(x) = f(g(x)) = f(|x|) = \frac{1}{|x|}$$

domain of g is $(-\infty, 0]$
range of g is $[0, \infty)$
domain of f $(0, \infty)$

- intersection is $(0, \infty)$.
- The part of domain of g that leads to this part of range of g is $(-\infty, 0)$.
- The domain of $f \circ g$ is $(-\infty, 0)$.

So $(f \circ g)(x) = \frac{1}{|x|} = -\frac{1}{x}$

since domain is $(-\infty, 0)$.



$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \left|\frac{1}{x}\right| = \frac{1}{|x|}$$

domain of f is $(0, \infty)$
range of f is $(0, \infty)$
domain of g $(-\infty, 0]$

→ intersection is empty.
⇒ The domain of $g \circ f$ is empty.

So even though $(g \circ f)(x)$ and $(f \circ g)(x)$ look the same, they are different functions because they have different domains.

For example

$(f \circ g)(-2) = f(g(-2)) = f(2) = \frac{1}{2}$.

$(g \circ f)(-2) = g(f(-2))$ = does not exist since $f(-2)$ does not exist.