

Instructions:

- Attempt all questions.
- The test is out of 100 marks.
- There are 8 questions:
Questions 1–6 are 10 marks each
Questions 7–8 are 20 marks each.
- You have 65 minutes to complete the test.
- You may use calculators on this test.

Advice:

- Budget your time.
- Do questions which you know how to do immediately first.
- Leave questions which you find difficult until last.
- Ask for clarification if you do not understand a question.
- You must show your work. Label sketches well.

Problem 1. (10 marks) Assuming x , y , and z are positive, use properties of logarithms to write the expression as a single logarithm.

$$2 \ln(xy) + 5 \ln(y^2 z) - \ln(z y)$$

Problem 2. (10 marks) Find the algebraic expression for the inverse function $f^{-1}(x)$ if $f(x) = -2 \ln(-4x)$.

Problem 3. (10 marks) Solve the equation $\frac{12}{1 - 4e^{-3x}} = e^{3x}$ algebraically for x .

Problem 4. (10 marks) Given $f(x) = \sqrt{x+9}$, simplify the quantity $\frac{f(x+h) - f(x)}{h}$ as much as possible. You should simplify until substituting zero for h will not yield an indeterminate form.

Problem 5. (10 marks) Sketch the function $f(x) = -e^{1+x}$ and label one (x, y) pair on the sketch.

Problem 6. (10 marks) Given $f(x) = \ln(\sqrt{x})$, $g(x) = e^{x/4}$, and $h(x) = x^2$. Find the composition $(h \circ g \circ f)(x)$ and simplify as much as possible. Your final answer should **not** have exponentials and logarithms in it.

Problem 7. (20 marks) Solve the equation $\ln(x + 10) + \ln x = 2$ algebraically. Be sure to eliminate any extraneous solutions.

Problem 8. (20 marks) Suppose 64 zombies are unleashed on a large city. Every 12 hours, the number of zombies in the city doubles. Find an expression for the number of zombies in the city t days after the zombies were unleashed. When will the number of zombies be 78976? (give the answer both exactly and as a decimal)

Show all your calculations. Derive any formulas you need, do not simply plug numbers into a population growth formula you have memorized.