

**Name:**

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**Instructions:**

- Attempt all questions.
- The test is out of 100 marks.
- There are 7 questions:  
Questions 1–6 are 10 marks each  
Questions 7–8 are 20 marks each.
- You have 65 minutes to complete the test.
- You may use calculators on this test.

**Advice:**

- Budget your time.
- Do questions which you know how to do immediately first.
- Leave questions which you find difficult until last.
- Ask for clarification if you do not understand a question.
- You must show your work. Label sketches well.

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**Problem 1.** (10 marks) Assuming  $x$ ,  $y$ , and  $z$  are positive, use properties of logarithms to write the expression as a single logarithm.

$$2 \ln(xy) + 5 \ln(y^2z) - \ln(zy)$$

**Problem 2.** (10 marks) Find the algebraic expression for the inverse function  $f^{-1}(x)$  if  $f(x) = -2 \ln(-4x)$ .

**Problem 3.** (10 marks) Solve the equation  $\frac{12}{1 - 4e^{-3x}} = e^{3x}$  algebraically for  $x$ .

**Problem 4.** (10 marks) Given  $f(x) = \sqrt{x+9}$ , simplify the quantity  $\frac{f(x+h) - f(x)}{h}$  as much as possible. You should simplify until substituting zero for  $h$  will not yield an indeterminate form.

**Problem 5.** (10 marks) Sketch the function  $f(x) = -e^{1+x}$  and label one  $(x, y)$  pair on the sketch.

**Problem 6.** (10 marks) Given  $f(x) = \ln(\sqrt{x})$ ,  $g(x) = e^{x/4}$ , and  $h(x) = x^2$ . Find the composition  $(h \circ g \circ f)(x)$  and simplify as much as possible. Your final answers should **not** have exponentials and logarithms in them.

**Problem 7.** (20 marks) Solve the equation  $\ln(x + 10) + \ln x = 2$  algebraically. Be sure to eliminate any extraneous solutions.

**Problem 8.** (20 marks) Suppose 64 zombies are unleashed on a large city. Every 12 hours, the number of zombies in the city doubles. Find an expression for the number of zombies in the city  $t$  days after the zombies were unleashed. When will the number of zombies be 78976?  
Show all your calculations. Derive any formulas you need, do not simply plug numbers into a population growth formula you have memorized.