

Name:

Instructions:

- Attempt all questions.
- The test is out of 100 marks.
- There are 7 questions:
Questions 1–6 are 10 marks each
Questions 7–8 are 20 marks each.
- You have 65 minutes to complete the test.
- You may use calculators on this test.

Advice:

- Budget your time.
- Do questions which you know how to do immediately first.
- Leave questions which you find difficult until last.
- Ask for clarification if you do not understand a question.
- You must show your work. Label sketches well.

Problem 1. (10 marks) Assuming x , y , and z are positive, use properties of logarithms to write the expression as a single logarithm.

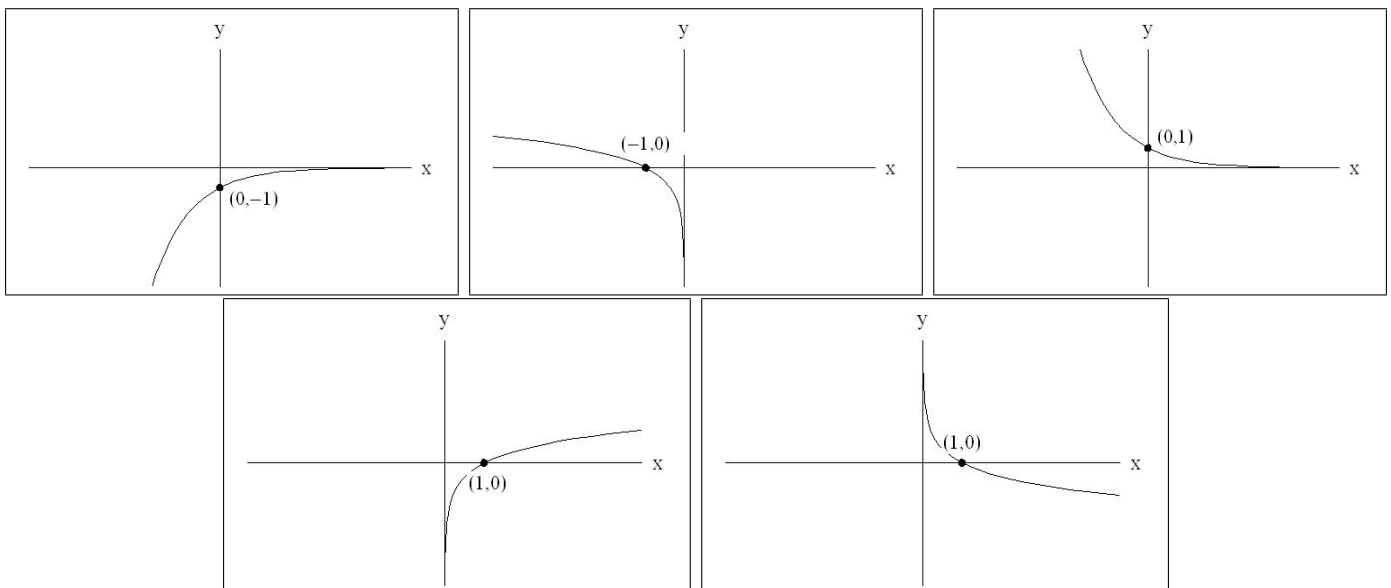
$$2 \ln(xy) + 5 \ln(yz^2) - \ln(zy)$$

Problem 2. (10 marks) Find the algebraic expression for the inverse function $f^{-1}(x)$ if $f(x) = \frac{1}{\ln(-2x)}$.

Problem 3. (10 marks) Solve the equation $\frac{1}{1 + be^{-x}} = 2$ algebraically for x (b is a constant which will remain in your solution).

Problem 4. (10 marks) Match the functions to their graphs. You do not need to show any work for this problem.

- a) $y = \ln x$ b) $y = \ln(-x)$ c) $y = -\ln x$ d) $y = -e^{-x}$ e) $y = e^{-x}$



Problem 5. (10 marks) Sketch the function $f(x) = -e^{1+x}$ and label one (x, y) pair on the sketch.

Problem 6. (10 marks) Given $f(x) = \ln(\sqrt{x})$, $g(x) = e^{x/4}$, and $h(x) = x^2$. Find the composition $(f \circ g \circ h)(x)$ and simplify as much as possible. Your final answers should **not** have exponentials and logarithms in them.

Problem 7. (20 marks) Solve the equation $\ln(x + 10) + \ln x = 2$ algebraically. Be sure to eliminate any extraneous solutions.

Problem 8. (20 marks) Suppose a culture of 100 bacteria is put in a petri dish and the culture triples in size every 12 hours. Find an expression for the number of bacteria present at any time t hours later. At what time will the number of bacteria be 10^3 ? Show your work.