

Name:

Instructions:

- Attempt all questions.
- The test is out of 100 marks.
- There are 10 questions worth 10 marks each.
- You have 65 minutes to complete the test.

Advice:

- Budget your time—spend about 5 minutes on each problem.
- First, do those questions which you know how to do immediately.
- Leave questions which you find difficult until last.
- Ask for clarification if you do not understand a question.
- You must show your work. Label sketches well (points of interest, asymptotes, axes, etc).

Good Luck!

Problem 1. (10 marks) For the quadratic function $f(x) = 14x^2 - 3x + 47$, convert to the vertex form $f(x) = a(x - h)^2 + k$ by completing the square. Identify the vertex and axis of symmetry for this quadratic function. You must use completing the square in this problem.

Problem 2. (5+5=10 marks) Given the function $g(x) = -7(2x - 8)^5(3x + 10)^2$.

(a) State the degree of the polynomial, and the zeros with their multiplicity.

(b) Describe the end behaviour of this function.

Problem 3. (10 marks) Find the remainder $r(x)$ when $g(x) = -2x^3 - 2$ is divided by $d(x) = x - 4$ using long division of polynomials.

Problem 4. (10 marks) Identify all possible rational factors of the polynomial $f(x) = 5x^5 - 8x^3 + 10x - 3$. Explain how you would determine which numbers from your list are actually going to be a factor (you don't have to find the actual factors, just explain the process).

Problem 5. (10 marks) Sketch the polynomial $f(x) = (x - 1)^3(x + 2)^2$ by hand. Show all your work.

Problem 6. (10 marks) Sketch the function $g(x) = 5 + \frac{2}{x+3}$ by transforming $y = f(x) = \frac{1}{x}$.

Problem 7. (10 marks) Sketch the rational function $h(x) = \frac{(3x-12)^2}{x}$ by hand (find x -intercepts, vertical asymptotes, and end behaviour).

Problem 8. (10 marks) Solve the inequality $\frac{1}{x+1} + \frac{1}{x-3} \leq 0$.

Problem 9. (10 marks) The volume of an enclosed gas (at a constant temperature) varies inversely as the pressure. This means the relationship between volume V and pressure P can be written as

$$V = \frac{k}{P} \qquad \text{(Boyle's Law)}$$

where k is the proportionality constant.

If the pressure of a 3.46 L sample of neon gas at 302° K is 0.926 atm, what would the volume be at a pressure of 1.452 atm if the temperature does not change?

Problem 10. (10 marks) Given the function $f(x) = ax^2 + bx + c$, find (simplify as much as possible, until the denominator equals 1) the average rate of change of the function over the interval $[x_0, x_0 + h]$, which is given by

$$\frac{f(x_0 + h) - f(x_0)}{h}$$