

**Instructions:**

- Attempt all questions.
- The test is out of 100 marks.
- There are 10 questions, 10 marks each.
- You have 60 minutes to complete the test.
- You may use calculators on this test.

**Advice:**

- Budget your time.
  - Do questions which you know how to do immediately first.
  - Leave questions which you find difficult until last.
  - Ask for clarification if you do not understand a question.
  - You must show your work. Label sketches well.
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**Problem 1.** (5+5=10 marks)

(a) Determine the domain of the function  $f(x) = \frac{4}{\sqrt{8-5x}}$

(b) Given  $w(t) = \frac{45-8t^2}{\sqrt{8+4t^6}}$ , what function does the function  $w(t)$  approach as  $t \rightarrow \infty$ ? (end behaviour)

**Problem 2.** (10 marks) Draw a well labelled arrow diagram for the addition of two functions, where  $f$  has domain  $A$  and range  $B$  and  $g$  has domain  $D$  and range  $R$ :  $(f+g)(x) = f(x) + g(x)$ . Clearly indicate in your diagram what the domain of  $f+g$  is.

**Problem 3.** (10 marks) Given  $f(x) = \frac{x^2 - 2}{3}$ , simplify the quantity  $f(x + h) - f(x - h)$  as much as possible.

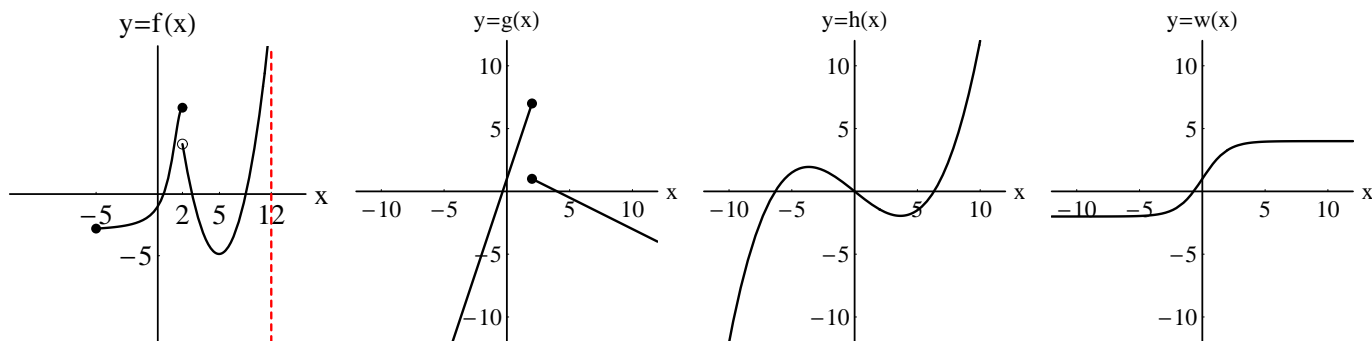
**Problem 4.** (10 marks) Determine whether the following function is even, odd, or neither. Use the algebraic technique to determine if a function is even or odd, rather than attempting to sketch the function.

$$g(x) = \frac{x^3 - x}{x^4 + 1}$$

**Problem 5.** (10 marks) Find a formula  $f^{-1}(x)$  for the inverse of the function (you do not have to discuss domain and range):

$$f(x) = \frac{1 + 5x}{3 - 2x}$$

**Problem 6.** (10 × 1 = 10 marks) Answer questions (i)–(x) based on the following graphs.



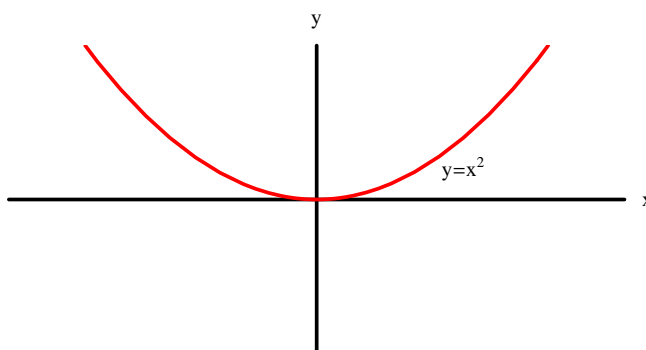
- (i)  $f(x)$  is continuous for  $x \in [2, 12]$  ..... T F
- (ii)  $f(x)$  has a vertical asymptote given by  $y = 12$  ..... T F
- (iii)  $f(x)$  is a function with domain  $x \in [-5, 12)$  ..... T F
- (iv)  $f(-5) = 5$  ..... T F
- (v)  $g(x)$  is not a function ..... T F
- (vi)  $h(-x) = -h(x)$  ..... T F
- (vii)  $h(x)$  is an even function ..... T F
- (viii)  $h(x)$  is a one-to-one function ..... T F
- (ix)  $w(x)$  is bounded above and bounded below ..... T F
- (x)  $w(x)$  has two horizontal asymptotes ..... T F

**Problem 7.** (10 marks) Sketch the graph of the piecewise defined function  $f$ , and label three  $(x, y)$  ordered pairs on the graph. From your graph, what is the range of  $f$ ?

$$f(x) = \begin{cases} -2 - 2x & \text{if } x > 0 \\ -|x| & \text{if } x \leq 0 \end{cases}$$

**Problem 8.** (10 marks) Given the functions  $f(x) = x^4 - x^2$  and  $g(x) = \sqrt{x^{3/2} - x}$ , determine the composition  $(f \circ g)(x)$  (simplify as much as possible). You do not have to discuss domains.

**Problem 9.** (10 marks) Given below is a sketch of the function  $f(x) = x^2$ . Using what we've learned about translating a graph, draw a sketch of the function  $g(x) = (x - 1)^2 + 4$  (you can add it directly to the sketch below if you like).



**Problem 10.** (10 marks) Given below is a sketch of the function  $y = f(x)$ . Add to this a sketch of the inverse function  $y = f^{-1}(x)$ . Label one  $(x, y)$  ordered pair on the inverse function.

