Example 2.4.32 Using only algebraic methods, find the cubic function with the given table of values. Check by sketching.

x	f(x)
-2	0
-1	24
1	0
5	0

A cubic polynomial has four coefficients, $f(x) = ax^3 + bx^2 + cx + d$. Since we are given the three zeros of the polynomial, we can write the polynomial in factored form, with only one coefficient left to determine:

$$f(x) = a(x - c_1)(x - c_2)(x - c_3) = a(x - (-2))(x - 1)(x - 5) = a(x + 2)(x - 1)(x - 5)$$

The fourth point given can be used to determine the coefficient a:

$$\begin{aligned} f(x) &= a(x+2)(x-1)(x-5) \\ f(-1) &= a((-1)+2)((-1)-1)((-1)-5) = 24 \\ a(1)(-2)(-6) &= 24 \\ a(12) &= 24 \\ a &= \frac{24}{12} = 2 \end{aligned}$$

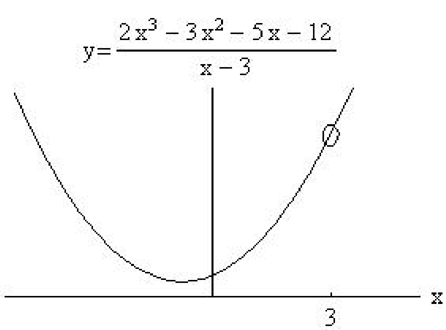
The cubic polynomial passing through the points is $f(x) = 2(x + 2)(x - 1)(x - 5)$.

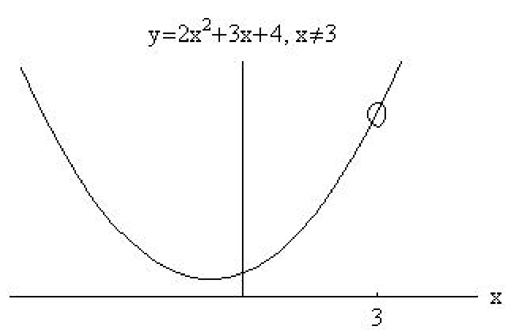
Example 2.4.74 Graph the two functions

$$f(x) = \frac{2x^3 - 3x^2 - 5x - 12}{x - 3}, \qquad g(x) = 2x^2 - 3x + 4, x \neq 3.$$

How are these functions related? Include a discussion of the domain and continuity of each function.

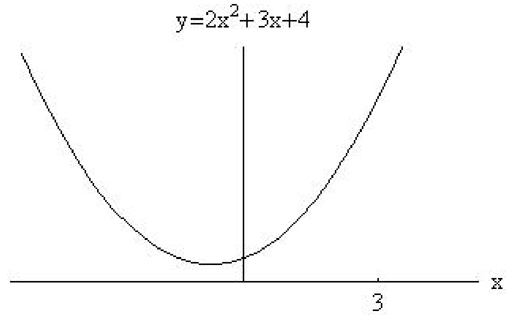
I graphed these functions using a computer.





These functions are exactly the same. They both have domain $x \in (-\infty, 3) \cup (3, \infty)$ and are discontinuous at $x = 3$ (the open circle on the sketch indicates the discontinuity).

Compare the above with the function $h(x) = 2x^2 - 3x + 4$:



which is different from f and g since it has domain $x \in (-\infty, \infty)$ and is continuous for all x.