e  We define \( X_i \) such that \( X_i = 1 \), for treatment \( i \); 0, otherwise, where \( i = 1, 2, 3, 4 \). Then the appropriate regression model is \( Y = \beta_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + E \) where the regression coefficients are as follows:

\[
\beta_0 = \mu_5, \quad \alpha_1 = \mu_1 - \mu_5, \quad \alpha_2 = \mu_2 - \mu_5, \quad \alpha_3 = \mu_3 - \mu_5, \quad \alpha_4 = \mu_4 - \mu_5
\]

For \( X_i = -1 \), for treatment 5; 1, for treatment \( i, (i=1,2,3,4) \); 0, otherwise.

The regression coefficients are:

\[
\beta_0 = \mu, \quad \alpha_1 = \mu_1 - \mu, \quad \alpha_2 = \mu_2 - \mu, \quad \alpha_3 = \mu_3 - \mu, \quad \alpha_4 = \mu_4 - \mu, \quad \text{and also}
\]

\[-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) = \mu_5 - \mu \]

f  Calculating by hand: We rank the sample means in descending order of magnitude:

\( \bar{Y}_1 > \bar{Y}_5 > \bar{Y}_4 > \bar{Y}_2 > \bar{Y}_3 \)

So the order of comparisons to be made is 1 vs 3, 1 vs 2, 1 vs 4, 1 vs 5, 5 vs 3, 5 vs 2, 5 vs 4, 4 vs 3, 4 vs 2, 2 vs 3.

\[\text{MSE} = 2.34 \text{ (from the ANOVA table in (b)), and} \]

\[n_i = 6 = n^* (i = 1, 2, 3, 4, 5); \quad n = \sum_{i=1}^{5} n_i = 30 \]

\[k = 5, \quad \alpha = 0.05 \]

\[\text{Scheffé's method:} \]

\[S^2 = (k-1) F_{k-1, n-k, 1-\alpha} = 4 * F_{4, 25, 0.95} = 4 * 2.75 = 11 \]

\[S = 3.317 \]

Thus the half width, \( w_s \), by Scheffé’s method is

\[w_s = S \sqrt{\frac{\text{MSE}}{\frac{1}{6} + \frac{1}{6}}} = 3.317 \sqrt{2.34 \left(\frac{1}{3}\right)} = 2.929 \]

\[\text{Tukey's method:} \]

\[T = \frac{1}{\sqrt{n^*}} q_{k,n-k,1-\alpha} = \frac{1}{\sqrt{6}} q_{5,25,0.95} = \left(\frac{1}{\sqrt{6}}\right)(4.158) = 1.697 \]

Thus the half width, \( w_T \), by Tukey’s method is:

\[w_T = T \sqrt{\text{MSE}} = (1.697) \sqrt{2.34} = 2.596 \]

\[\text{Bonnferroni's method:} \]

There are \( C_2^4 = 10 \) pairwise comparisons; so \( \alpha^* = \frac{\alpha}{10} = 0.005 \).

The half width by Bonnferoni’s method is \( w_B = t_{25,0.9975} \sqrt{\frac{\text{MSE}}{\frac{1}{6} + \frac{1}{6}}} = 2.827 \)