Please choose the best answer out of the choices given. (each 4 pts)

1. The heights (in inches) of males in the U.S. are believed to be normally distributed with mean $\mu$. The average height of a random sample of 25 American adult males is found to be $\bar{x} = 69.72$ inches and standard deviation of the 25 heights is found to be $s = 4.15$. The standard error of $\bar{x}$ is

$$SE(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{4.15}{\sqrt{25}} = 0.83$$

A) 0.17  B) 0.69  C) 0.83  D) 2.04

2. A simple random sample of five female basketball players is selected. Their heights (in cm) are 170, 175, 169, 183, and 177. What is the standard error of the mean of these height measurements?

$$SE(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{5.675}{\sqrt{5}} = 2.538$$

A) 2.538  B) 2.837  C) 5.075  D) 5.675

3. Other things being equal, the margin of error of a confidence interval increases as

A) the sample size increases.

B) the confidence level decreases.

C) the population standard deviation increases.

D) none of the above.

Use the following to answer questions 4-5:

The heights of a simple random sample of 400 male high school sophomores in a Midwestern state are measured. The sample mean is $\bar{x} = 66.2$ inches. Suppose that the heights of male high school sophomores follow a normal distribution with standard deviation $\sigma = 4.1$ inches.

4. What is a 95% confidence interval for $\mu$?

A) (58.16, 74.24)

B) (59.46, 72.94)

C) (65.80, 66.60)

D) (65.86, 66.54)

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}} = 66.2 \pm 1.96 \cdot \frac{4.1}{\sqrt{400}} = 66.2 \pm 0.4018 = (65.7982, 66.6018)$$
5. If a 95% confidence interval was constructed instead, how would the margin of error compare to the one used to create the 90% confidence interval?

A) The margin of error for the 95% confidence interval would be smaller.
B) The margin of error for the 95% confidence interval would be the same.
C) The margin of error for the 95% confidence interval would be larger.
D) This cannot be determined from the information given.

6. The hypotheses \( H_0: \mu = 350 \) versus \( H_a: \mu < 350 \) are examined using a sample of size \( n = 20 \). The one-sample \( t \) statistic has the value \( t = -1.88 \). What do we know about the \( P \)-value of this test?

A) \( P \)-value < 0.01    B) 0.01 < \( P \)-value < 0.025    C) 0.025 < \( P \)-value < 0.05    D) \( P \)-value > 0.05

7. Scores on the SAT Mathematics test are believed to be normally distributed. The scores of a simple random sample of five students who recently took the exam are 550, 620, 710, 520, and 480. What is a 95% confidence interval for \( \mu \), the population mean score on the SAT Math test?

A) (456.7, 695.3)    B) (463.4, 688.6)    C) (480.8, 671.2)    D) (496.5, 655.5)

Use the following to answer questions 8-10:

A simple random sample of 100 postal employees is used to test if the average time postal employees have worked for the postal service has changed from the value of 7.5 years recorded 20 years ago. The sample mean was \( \bar{x} = 7 \) years with a standard deviation of \( s = 2 \) years. Assume the distribution of the time the employees have worked for the postal service is approximately normal. The hypotheses being tested are \( H_0: \mu = 7.5 \), \( H_a: \mu \neq 7.5 \). A one-sample \( t \) test will be used.

8. What are the appropriate degrees of freedom for this test?
A) 7    B) 19    C) 99    D) 100

9. What is the \( P \)-value for the one-sample \( t \) test?
A) \( P \)-value < 0.01    B) 0.01 < \( P \)-value < 0.05    √    C) 0.05 < \( P \)-value < 0.10    D) \( P \)-value > 0.10

10. What is a 95% confidence interval for \( \mu \), the population mean time the postal service employees have spent with the postal service?
A) 7 ± 2    B) 7 ± 1.984    C) 7 ± 0.4    D) 7 ± 0.2

11. When can the pooled two-sample \( t \) procedure be used?

(A) When two simple random samples are taken from two populations with the same mean.
(B) When two simple random samples are taken from two populations with the same standard deviation.
(C) When two simple random samples are taken from two populations with different standard deviations.

12. When calculating a two-sample \( t \) confidence interval, which level of confidence will give the largest interval?

(A) 80\%  (B) 90\%  (C) 95\%  (D) 100\%

13. A test of the null hypothesis \( H_0 : \mu = \mu_0 \) gives test statistic \( z = 1.9 \).

(a) What is the P-value if the alternative is \( H_a : \mu < \mu_0 \) ? (4 pts)

(b) What is the P-value if the alternative is \( H_a : \mu \neq \mu_0 \) ? (4 pts)

\[
P-value = 2 \times 0.0287 = 0.0574
\]

14. The one-sample \( t \) statistic from a sample of \( n=30 \) observations for the two-sided test of

\( H_0 : \mu = 50 \) and \( H_a : \mu \neq 50 \)

has the value \( t=1.82 \).

(a) What are the degrees of freedom for \( t \) ? (4 pts)

\[
df = 30 - 1 = 29
\]

(b) How would you report the P-value for this test? (4 pts)

(c) Is the value \( t=1.82 \) statistically significant at the 5\% level? (4 pts)

\[
\text{Since } p-value > 0.05, \\
\text{We do not reject } H_0 \text{ at the 5\% level.}
\]

"No" it is not statistically significant at the 5\% level.
15. The table below gives the pretest and posttest scores on the MLA listening test in Spanish for 10 high school Spanish teachers who attended an intensive summer course in Spanish.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Diff</th>
<th>(Posttest - Pretest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>29</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>30</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>31</td>
<td>32</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>30</td>
<td>4</td>
<td></td>
</tr>
<tr>
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<td>20</td>
<td>16</td>
<td>-4</td>
<td></td>
</tr>
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<td>30</td>
<td>25</td>
<td>-5</td>
<td></td>
</tr>
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<td>34</td>
<td>31</td>
<td>-3</td>
<td></td>
</tr>
<tr>
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<td>15</td>
<td>18</td>
<td>3</td>
<td></td>
</tr>
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<td>28</td>
<td>33</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>25</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(a) We hope to show that attending the institute improves listening skills. State an appropriate $H_0$ and $H_a$. Be sure to identify the parameters appearing in the hypotheses. (4 pts)

$H_0: \mu_{diff} = 0$ vs. $H_a: \mu_{diff} > 0$

(b) Carry out a test. Can you reject $H_0$ at the 5% significance level? (8 pts)

$\bar{x} = 0.7$

$s = 3.94$

$n = 10$

$df = 9$

$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{0.7 - 0}{3.94 / \sqrt{10}} = 0.59$

Since $p-value > 0.15$

We do not reject $H_0$ at the 5% level.

16. Pat wants to compare the cost of one- and two-bedroom apartments in the area of your campus. She collects data for a random sample of 5 advertisements of each type. Here are the rents for the two-bedroom apartments (in dollars per month):

595, 500, 580, 650, 670

Here are the rents for the one-bedroom apartments:

500, 650, 600, 505, 460

Pat wonders if two-bedroom apartments rent for significantly more than one-bedroom apartments.

(a) State appropriate null and alternative hypotheses. (4 pts)

$H_0: \mu_1 = \mu_2$ vs. $H_a: \mu_1 > \mu_2$

(b) Report the test statistic and the P-value. What do you conclude? (8 pts)

$df = \min(4, 4) = 4$

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{599 - 543}{\sqrt{\frac{66.54^2}{5} + \frac{58.89^2}{5}}} = 1.21$

Since $p-value < 0.05$

We reject $H_0$ at the 5% level.
17. A political analyst has curious if younger adults were becoming more conservative. He decided to see if the mean age of registered Republicans was lower than that of registered Democrats. He selected an SRS of 128 registered Republicans from a list of registered Republicans and determined the mean age to be \( \bar{x}_1 = 39 \) years with a standard deviation \( s_1 = 8 \) years. He also selected an independent SRS of 200 registered Democrats from a list of registered Democrats and determined the mean age to be \( \bar{x}_2 = 40 \) years with a standard deviation \( s_2 = 10 \) years. Let \( \mu_1 \) and \( \mu_2 \) represent the mean ages of the populations of all registered Republicans and Democrats, respectively. Suppose that the distributions of age in the populations of age in the populations of registered Republicans and of registered Democrats have the same standard deviation. Assuming two-sample t procedures are safe to use.

(a) Calculate the pooled estimator \( s_p^2 \) of \( \sigma^2 \). (4 pts)

\[
\begin{align*}
\bar{x}_1 &= 39 \\
\bar{x}_2 &= 40 \\
s_1 &= 8 \\
s_2 &= 10 \\
\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} &= \frac{(128 - 1)8^2 + (200 - 1)10^2}{128 + 200 - 2} = 85.9158
\end{align*}
\]

(b) Find a 90% confidence interval for \( \mu_1 - \mu_2 \). (4 pts)

\[
\begin{align*}
\bar{x}_1 - \bar{x}_2 &\pm t^{*} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \\
t^{*} &= t_{326,90} \\
&= t_{130,90} \\
&= 1.660
\end{align*}
\]

\[
\begin{align*}
39 - 40 &\pm 1.660 \sqrt{85.9158 \cdot \frac{1}{128} + \frac{1}{200}} \\
&= -1 \pm 1.9423 \\
&= (-1 - 1.9423, -1 + 1.9423) \\
&= (-2.9423, 0.9423)
\end{align*}
\]